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EXPERIMENTS

ON

SCHISTOSITY AND SLATY CLEAVAGE

BY

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DEPARTMENT OF THE INTERIOR,
UNITED STATES GEOLOGICAL SURVEY,
Washington, D. C., June 4, 1904.

SIR: I have the honor to transmit herewith a paper by myself describing experiments made for the purpose of testing disputed theories of schistosity and slaty cleavage.

Very respectfully,

GEORGE F. BECKER,
Geologist in Charge Division of Chemical and Physical Research.

HON. CHARLES D. WALCOTT,
Director United States Geological Survey.

EXPERIMENTS ON SCHISTOSITY AND SLATY CLEAVAGE.

By George F. Becker.

Abundance of fissile rocks.—A very large part of the rocks exposed at the earth's surface exhibit schistosity or cleavability not ascribable to sedimentation and often well marked in masses of igneous origin. In many cases the surfaces along which rocks cleave intersect one another, producing a foliated structure. In other instances the cleavage occurs in nearly parallel planes or along surfaces having large radii of curvature. None of these phenomena are confined to highly indurated rocks, as might be inferred from some discussions, but are found well developed in soft and fragile unmetamorphosed shales, as all who have made observations in disturbed Tertiary areas are aware. Specimens of these shales are so fragile, however, that they are scantily represented in collections. Slaty cleavage, as the term is used in this paper, means simply the most regular and extreme form of cleavability or schistosity, in which the laminæ are thin and are bounded by substantially parallel surfaces, no matter whether the material exhibiting these properties is indurated, as in the valuable roofing slates, or is soft and fragile, as in the comparatively worthless shales. Though argillaceous rocks exhibit the most perfect cleavage, this does not apparently differ in kind from the schistosity sometimes found in grit beds, limestone, granite, quartzite, and basic eruptives. A structure which is at least analogous is found in rolled or drawn metals, in terra-cotta articles, such as roofing tiles, and in pastry.[a] In short, slaty cleavage is a structure, or a group of structures not readily distinguishable from one another, and has a character of its own independent of the material which may exhibit it, although some substances are better fitted than others to display it in perfection. This relationship is recognized in the French term "schiste," which, in Gallic usage, comprehends shales, slates, and the crystalline schists of English writers. In English usage schist is commonly synonymous with crystalline schist, but the frequent use of this adjective would seem to imply the existence of schists which are not crystalline, and these could hardly be defined otherwise than as equivalent to shales.

[a] Cf. Daubrée, A., Géol. Exp., pp. 398 and 428.

Slaty cleavage, even in highly indurated rock, passes over by insensible gradations into less simple forms of schistosity, and vast masses of the crystalline schists show cleavage planes in systems intersecting one another at acute angles. It has always seemed to me that a true explanation of cleavage must include the theory of foliated schists as well as of roofing slate. The association of mica with cleavable rocks has often been insisted upon, perhaps with too great emphasis. Some shales contain much mica, but others contain only a little. According to Professor Rosenbusch, shales (Schieferthone) have the same mineralogical composition as clays, and mica is not an essential constituent. Sorby mentions clay slates which carry very little mica, and G. W. Hawes[a] describes a roofing slate from Littleton, N. H., which on microscopic examination is found to consist of fragments of quartz and feldspar as fine as dust, although in the larger part of the rocks called clay slate in New Hampshire he found abundant mica. Now clay slates and some shales have as good cleavage as mica-schists. Again, quartz-schists and other cleavable rocks contain very little mica. The grit beds or sandy strata found in slates do not always contain much mica, and yet their cleavage is manifestly of the same origin as that of the slate in which they are embedded. It is well known that near intrusive masses mica-schist not infrequently passes over into ordinary schists, and these into phyllites and clay slates, as if the amount of mica were characteristic of the degree of metamorphism rather than an index of the cleavability. "Compression," says Sir Archibald Geikie, "may give rise to slaty cleavage. But it has often been accompanied or followed by further internal transformations in the rocks. Chemical reactions have been set up and new minerals have been formed." In the study of slates it is often manifest that a portion of the mica is secondary. Thus on blind joints (Ausweichungsclivage of Heim) large continuous sheets of mica are frequently found. Muscovite is also well known to be one of the chief decomposition products of the feldspars, an alteration which is readily intelligible from a chemical standpoint. The increase of the mica content of phyllites as compared with shales seems to me most reasonably accounted for as a concomitant of the genesis of cleavage, though not an essential one.

Importance of cleavage.—Schistosity as a structure is important, and it is a part of the business of geologists to explain its origin. Slaty cleavage has further and greater importance as a possible tectonic feature. Scarcely a great mountain range exists, or has existed, along the course of which belts of slaty rock are not found, the dip of the cleavage usually approaching verticality. Are these slate belts equivalent to minutely distributed step faults of great total throw, or do they indicate compression perpendicular to the cleavage without attendant

[a] Hitchcock, C. H., Geology of New Hampshire, vol. 3, 1878, pt. 4, pp. 237, 238.

relative dislocation? Evidently the answer to this question is of first importance in the interpretation of orogenic phenomena.

Theories of cleavage.—The earliest theory of slaty cleavage assimilated it to mineral cleavage, a view not tenable after microscopic study. Mr. John Phillips in 1843 was the first to interpret slaty cleavage as an effect of mechanical strain. Mr. Daniel Sharpe in 1849 offered the explanation now most generally accepted by geologists, viz, that a fracture perpendicular to the line of pressure would run along the flattest faces of the component grains and meet the smallest number of them—a theory implying that the mass is heterogeneous and that the adhesion between the component particles is smaller than the cohesion within the particles. Dr. H. C. Sorby later described a variety of cleavage due to the presence of numerous microscopic blind joints.[a] Professor Tyndall in 1856 made exceedingly interesting experiments on this subject, obtaining admirable cleavage in wax. He denied that heterogeneity aided cleavage. In his first paper he asserted most emphatically that the cleavage was perpendicular to the direction of pressure, but in a footnote and in a later paper he indicated decided doubt as to this perpendicularity. Professor Daubrée (1879) also dissented from the view that heterogeneity is essential to slaty cleavage, and ascribed this structure to gliding ("glissement," slide) in the mass, which is equivalent to a denial of the perpendicularity of the causative force to the consequent cleavage.[b] In 1893 I published a theory founded on experiment and analysis. It is in agreement with Daubrée's idea, but more precise. According to this theory, cleavage is due to a weakening of cohesion[c] along planes of maximum tangential strain (or maximum slide). It is susceptible of proof[d] that deformation due to pressure is actually effected by relative motion of the mass in opposite directions parallel to these planes. When this movement exceeds the elastic limit and falls short of the breaking strain, it would seem inevi-

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[a] Sorby's theory of slate was that the preliminary effect of pressure on argillaceous strata is to give the mica an irregular distribution, and the final effect to rearrange the mica in new parallel planes. This hypothesis is still accepted in some text-books. To me it appears too fanciful for serious discussion. If mica scales, in all possible orientations, were to be mingled with mud, their average inclination to any plane would be 32° 42′ (or the well-known "average latitude of all places north of the equator"). If such a mass were to be compressed until the average angle were only 2° to a given plane, the thickness of the mass must be reduced to one-eighteenth. If a sediment containing mica were to be treated according to Sorby's theory, it would seemingly be needful to press it at first in such a way as to increase its thickness some eighteen times and then to compress it again in another direction to about its original thickness.

The attempt has been made to account for the cleavage surfaces on Sorby's hypothesis as maximum cleavability. I can not concur in this view. The cleavage in slate is confined within very narrow limits, perhaps one degree. Slate may be broken indeed at greater inclinations, but the ruptured surfaces do not then show imperfect cleavage; they are conchoidal or irregular and destitute of cleavage.

[b] The reader will find a digest of the literature in Mr. Alfred Harker's memoir on slaty cleavage, Brit. Assoc., 1885, p. 813, and some further notes in my paper on finite homogeneous strain, flow, and rupture of rocks, Bull. Geol. Soc. America, vol. 4, 1893, pp. 75–87.

[c] As H. Rogers put it, "the cohesive force is obviously at a minimum of intensity in the direction perpendicular to these planes" of cleavage. Trans. Royal Soc., Edinburgh, vol. 21, 1857, p. 450.

[d] See note on the theory of slaty cleavage appended to this paper.

table that the cohesion should be diminished and that cleavage should result.[a] Only in ideally brittle substances is there no interval between the elastic limit and the breaking strain. It is of course certain that the material constituting slate has been strained far beyond its elastic limit; and that it has a breaking strain is often manifested by blind joints. These planes stand at an angle of 45° or more to the direction of greatest local linear compression. There are at least two sets of them, and maybe four, symmetrically disposed with reference to this direction. In cases of double cleavage—so usual in disturbed areas, both in distinct development and in the more or less irregular form of ordinary schistosity and foliation—cleavage is produced on both sets of planes, so that a cross section shows acute-angled rhombs somewhat like those indicated in fig. 13, Pl. III, and in fig. 30, Pl. VI. In the case of forces acting on a supported mass at an acute angle to the plane of support, it was shown in my paper that the effect of viscosity would be to suppress all but one set of cleavages and to accentuate this remaining one. Tyndall's experiment, properly considered, was shown to be a case of this kind, and it was maintained that the cleavage of roofing slate is thus to be explained. If this be true, a belt of slate is equivalent to a great fault distributed over an infinite number of infinitesimal steps.[b]

Distinction between Sharpe's theory and mine.—The distinction between Sharpe's theory and mine is well defined. If in any portion of the mass before strain a small sphere is supposed to be marked out, this sphere after strain will have become an ellipsoid, called the strain ellipsoid. If Sharpe's theory is correct, the cleavage due to pressure will be in surfaces perpendicular to the smallest axis of the strain ellipsoid. If my theory is correct, the cleavage will make with this smallest axis an acute angle equal to or greater than 45°, and increasing as the strain grows greater.

Means of studying the strain ellipsoid.—The general nature of the experiments needed to compare the theories is made plain by this contrast. It amounts to a study of the strain ellipsoid. One means to this end is to incorporate into a mass to be experimented upon small spheres of the same material as the remainder, but of a distinguishable color. After straining is effected dissection or rupture, by exposing the distorted sphere, will show the local character of the strain and the posi-

[a] A pertinent illustration of weakening of cohesion is afforded by bars of mild steel ruptured by tension. The most plausible a priori idea of rupture by tension is that it would occur in a plane perpendicular to the line of force; and in hard steels this mode of rupture is often observed. In mild steels, on the other hand, the surface of rupture is rough and granular, the grains approaching pyramidal forms. The aggregate surface of such a fracture is far in excess of that of the mean plane. Now, since the rupture must follow a surface of least resistance, the resistance per unit area on the pyramidal faces must have been much smaller than that on the plane surface perpendicular to the tension. This can be due only to a weakening of cohesion along the pyramidal faces.

[b] This theory embraces rupture as well as simple and double or multiple cleavage. So far as jointing and cleavage due to blind joints are concerned, it has been accepted by some geologists, who regard true cleavage as a distinct phenomenon.

tions of the axes of the strain ellipsoid. Numerous experiments of this description have been made for this paper, and some of them are illustrated in fig. 2, Pl. II. They are instructive, but not sufficiently so. If it were practicable to build up an adequate block of small spheres and to fill in the interstices uniformly with the same material differently colored, the mass after strain would show the character of the strain ellipsoid at uniformly spaced intervals. This is impracticable, but the result sought can be attained very approximately in another way. If a block of material be pierced with fine holes at regular intervals, forming in one plane a network of small squares such as is shown in fig. 5, Pl. II, and the holes be filled with coloring matter, then strain, followed by dissection, will show the figures into which the small squares have been distorted. If the squares were very small, the sides of the distorted figures would be nearly straight and parallel. It would then be easy by a geometrical construction to inscribe ellipses tangent to the four sides at their middle points, and these ellipses would accurately represent the section of the strain ellipsoid. Even if the sides were somewhat curved, the strain at the center of the curvilinear parallelogram would be very closely represented by an ellipse found by a rational system of interpolation, as will be described in a note appended to this paper. Experiments have been made in this way also, the distorted figures being photographed and photographically enlarged to a convenient scale for constructing the ellipses.

Linear compression.—Experiments by Tyndall's method are very simply and easily executed, but the resulting strain is highly complex and indeed could not be discussed as a problem of pure mechanics in the present state of knowledge of the transmission of energy in plastic bodies. By the means noted in the last paragraph such experiments can be sufficiently elucidated for an investigation of slaty cleavage.

Rolling.—Another known means of producing slaty cleavage is by rolling out a cake of suitable material as a cook prepares pastry, or by passing the mass between rolls. It is easy to prick holes perpendicular to the surface of the cake before rolling, fill them with pigment, and, after distortion, to dissect the mass. The nature of the effect is seen in fig. 4 of Pl. II, the originally vertical lines being drawn out into parabola-like curves which are vertical only at the apex. Cleavage may be developed especially near the upper and lower surfaces of the rolled mass, to which it is nearly parallel. Such experiments, however, give results which are less definite and less easily discussed than those given by Tyndall's method. The manner of applying the force and the degree of rolling affect the result, as a matter of course, and it seems difficult to establish a standard of conditions.

Scission engine.—It is evidently most desirable to make experiments by producing simple well-known strains which will or may lead to cleavage. For this purpose I designed what may be called a "scission

engine," a simple mechanism, shown without its cover in fig. 1, Pl. I. A block of material to be experimented upon is placed in the space B, the cover (provided with grooves to receive PP) is put on, screwed down onto pillars, and the driving screw set in motion. The bars F H revolve round the fixed pivots F, and the hinges H bend while the plates PP slide in their own planes. The area and volume of the space B remain constant and the strain produced is approximately a scission (or shearing motion). On four of the six faces, however, there is friction, which interferes somewhat, but not seriously, with the perfection of the strain.[a] If a circle is inscribed on the upper surface of the block before strain, it becomes approximately an ellipse showing the orientation of the strain ellipsoid. When the space B is thoroughly filled, no rupture within the mass is possible, the finite movement being distributed over an infinite number of planes; but if the space is not filled, complex strains result and rupture may occur. The engine gives about the unit strain, or produces a block with two angles of 45°.

Ceresin and its treatment.—The experiments to be described have been made principally with ceresin and a smooth clay such as is used by sculptors for modeling. I have tried white wax, which was the material employed by Tyndall, but white ceresin is preferable. This substance is refined ozocerite and consists of a mixture of paraffins. The material at my disposal melts at about 60° C., but some of the component paraffins solidify at a higher temperature, so that at 60° the cooling melt is a pasty mass, like thick oatmeal gruel. It contracts greatly in solidifying and should be cast at as low a temperature as practicable to prevent radial crystallization. Shavings of the solidified mass under the microscope show brilliant polarization colors, so that the mass is a crystalline solid. A particle fused on a slide and allowed to cool also shows high double refraction and exhibits a hypidiomorphic structure, analogous to that of semiporphyritic granites of excessively fine grain. Castings chilled in ice and salt and then broken with a hammer display a very fine-grained granular structure and somewhat conchoidal fracture, with no apparent radial crystallization.

The strong analogy between this solid and a rock is deserving of special emphasis. I can not see how deformations in masses of ceresin can possibly differ in character from those in the vastly less tractable crystalline rocks.

In carrying out Tyndall's experiment with ceresin I cast cylinders about 1½ inches in diameter and nearly 2 inches in length. After the cylinders were cold the ends were planed down until all trace of the pit due to contraction was removed. Before compressing them they remained in a thermostat for some hours at 35°, because at considerably lower temperatures they rupture or crumble too easily. Com-

[a] To obviate in some measure the effect of friction toward the center of the block, I made the space B 6 cm. long and only 4 cm. wide.

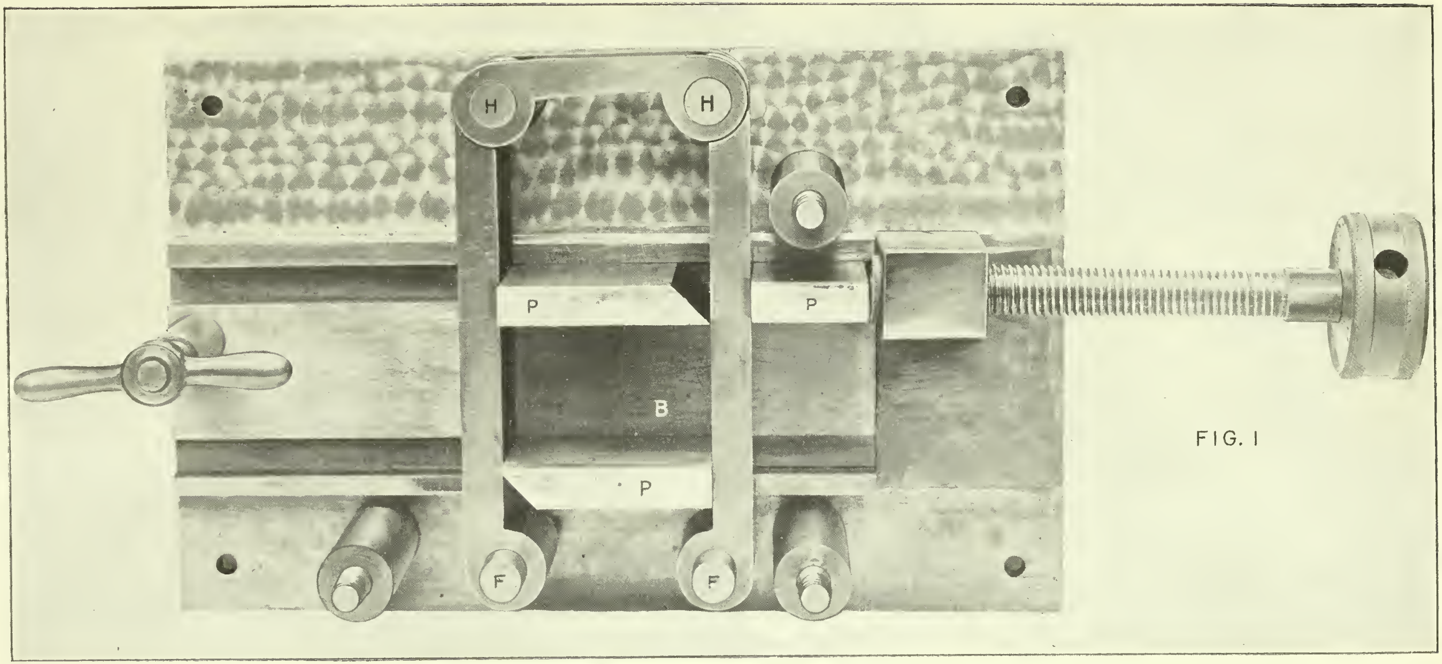

FIG. I

SCISSION ENGINE.

pression was effected in a copying press between two heavy glass plates mounted on boards. To avoid superficial chilling of the ceresin cylinders the glass plates were warmed to approximately the same temperature as the ceresin. Good cleavage is not obtained unless the cylinders are reduced to one-third of the original length, and more nearly perfect cleavage results from still further reduction. For accurate study the compressive force should be applied to the cylinders so slowly as not to rupture the edges of the cakes by tension. The compressed cakes were placed in a mixture of ice and salt for an hour or two, and then, being held edgewise on a small anvil, were broken by striking smart blows on the edge with a light hammer.

It is best to carry out such experiments in series. Some cylinders may be pierced with a network of fine holes (fig. 5, Pl. II) and, after compression, cut across in the plane of the perforations. To get the perforations sensibly in the same plane after compression, both piercing and squeezing must be carefully done. Other cylinders, of the same dimensions but not pierced, may be compressed to the same extent as the perforated cylinders and then, after chilling, split to show cleavage. If the cylinders are squeezed too rapidly the edges will split—a contingency to be avoided because the distribution of strain then becomes irregular and eludes systematic discussion.

For experiments on scission, blocks were planed to fit the opening B as accurately as possible and the blocks were given a temperature of about 20° before straining. They were cooled and broken as in the experiments on linear compression.

Experiments with clay.—The clay used burns to a pleasing terra-cotta color, without much shrinkage. Slides of the burnt mass show that the clay contains a large amount of finely divided quartz and a small amount of black mica in minute scales. There is also a trace of organic matter in this clay, for when first heated it blackens. For use, the clay should be moistened as little as is compatible with convenient modeling and kneading.

From a well-kneaded lump of clay it is easy to cut cylinders similar to those of ceresin described above. Excellent cleavage can be produced by compressing them in a press and burning the cakes lightly in an assay muffle furnace. Indeed, to get cleavage, it is sufficient to press a pellet, say a centimeter in diameter, under a spatula and toast it over a Bunsen lamp! Yet in some respects clay is inferior to ceresin for this experiment, because cylinders, after reduction to about five-eighths of their original diameter, crack at the edges, so that the strain can not be followed systematically by the method given above. Some of these cracks are meridional and due to tension, while others occur at about 45° to the line of force and are true joints on planes of maximum tangential strain. Such a cake of clay is shown in fig. 8, Pl. II. Precisely similar cracks are produced in steel cylinders subjected to

end pressure, and it is clear from these phenomena that the moist clay behaves mechanically like a true solid, relative motion of the particles taking place at an acute angle to the line of pressure. I have tried subdividing clay cylinders by a network of pin holes in one plane, and then compressing and dissecting them as described on a previous page. Up to the point where the edges begin to rupture, the deformation is exactly the same as in the cylinders of ceresin of similar dimensions and degree of compression.

On the other hand, clay behaves better than ceresin in the scission engine, apparently because I did not succeed in casting ceresin without small bubbles of included air, while it is easy to knead the clay until air is expelled. In the scission engine there is no tendency to cubical compression provided the space B is homogeneously filled, but this proviso is essential. In experiments on ceresin one acute angle of the space B is usually found empty, and though this space is never large, it is a disturbing condition. With well-kneaded clay the space B remains full after strain, and such blocks after burning show excellent cleavage.

Clay is a very instructive material to experiment upon for two reasons: Most or much of the natural slate is of argillaceous origin; and, again, the burning of the strained clay cakes may properly be considered as a true metamorphism, which nevertheless does not obliterate the cleavage mechanically induced.

Other materials tried.—Plaster of Paris pellets compressed under a spatula at just the right moment during the "setting" process and then dried out exhibit cleavage, a fact which is interesting because of the accompanying formation of selenite crystals. I have tried plaster paste in the scission engine repeatedly, but have not been able to hit exactly the right conditions. Air bubbles get into the liquid plaster while it is being poured into the space B, and if stress is applied too soon the plaster naturally sets without any development of cleavage. Moreover, plaster seems to begin to set from the outside, so that the space B was probably never homogeneously filled with a substance fitted to display the properties of homogeneous strain.

Lead cylinders pierced with holes and afterwards filled with tin wire and then compressed show by dissection just the same strains as do ceresin blocks. Such cases are shown in figs. 7 and 9, Pl. II. Similar results have been obtained with aluminium. I had hoped that these metals, cooled to the temperature of liquid air, would become brittle enough to exhibit cleavage, but was disappointed. The lead cakes seemed as tough as sole leather and I could not produce the least indication of a crack in the aluminium by the most vigorous use of the hammer.

Strain ellipsoids in ceresin.—In fig. 2, Pl. II, is shown a series of cakes of ceresin into which spherical pellets of ceresin tinged with a

Fig. 2.

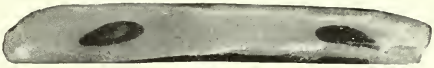

Fig. 3.

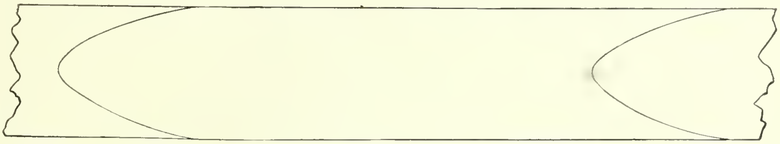

Fig. 4.

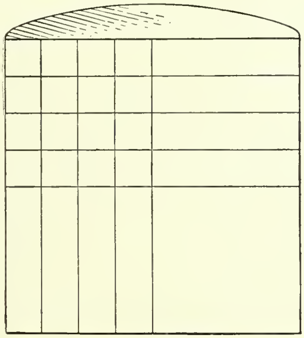

Fig. 5.

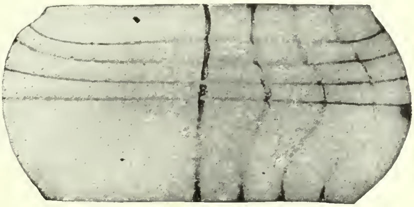

Fig. 6.

Fig. 7.

Fig. 8.

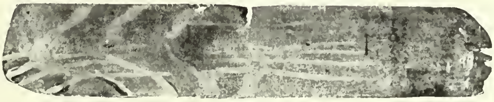

Fig. 9.

FIG. 2. STRAIN ELLIPSOIDS IN CERESIN.
FIG. 3. CLEFT STRAIN ELLIPSOID IN CERESIN.
FIG. 4. DIAGRAM OF DISTORTION IN ROLLING CLAY.
FIG. 5. DIAGRAM TO SHOW PIERCING OF CYLINDERS.
FIG. 6. CAKE OF CERESIN COMPRESSED TO TWO-THIRDS ORIGINAL HEIGHT.
FIG. 7. CAKE OF LEAD COMPRESSED TO TWO-THIRDS ORIGINAL HEIGHT.
FIG. 8. CAKE OF CLAY COMPRESSED TO TWO-THIRDS ORIGINAL HEIGHT, SHOWING PERIPHERAL RUPTURES.
FIG. 9. CAKE OF LEAD COMPRESSED TO ONE-THIRD ORIGINAL HEIGHT.

mere trace of vermilion were introduced during the process of casting. The cylinders after compression were cut radially to show the strain ellipsoids thus produced. If a composite photograph were to be taken of these and similar cases the position of the strain ellipsoid in most portions of the cross section would appear.

Many cakes containing pellets were cooled below the freezing point and split. These specimens show that the surfaces of cleavage are not parallel to the major axes of the strain ellipsoids. In some cases the cleavage developed by the hammer intersected the pellets, leaving no doubt whatever on this point (fig. 3, Pl. II). It is relatively seldom, however, that such a crack forms without splitting away a large part of the pellet and leaving some doubt as to the exact position of the major axis. Hence the observer is driven to a comparison between the dissected cakes without cracks and the split cakes. It seemed desirable, therefore, to devise a means of determining once for all the position of the strain ellipsoid in any and every part of the cake. This will now be described.

Construction of strain ellipsoids.—Fig. 5, Pl. II, is a diagram indicating the way in which cylinders were pierced with holes, forming a rectangular network covering a quarter of the cross section. A thread smeared with dry vermilion powder was drawn after the piercing needle and thus the interior of the perforations was coated with pigment.

Photographs of three cakes compressed after perforation and cut to show the network are shown on an enlarged scale in fig. 10, Pl. III. They are as nearly alike as the imperfection of the appliances used would permit. The central vertical lines in the cylinders were not absolutely central, nor were the plates between which the compression was effected accurately parallel planes. Hence, after compression the central line in each case is somewhat buckled.

From the middle one of these photographs a tracing was made, slightly modified by comparison with the other two, and then very much enlarged. In the diagram so procured ellipses were drawn by the methods explained at the end of this paper. The areas of the ellipses were next checked by a simple computation and found sensibly correct, showing that no considerable error had occurred in copying or construction. The value of the axes of the ellipses being found and the volume of the ellipsoids known, the planes of maximum tangential strain are also immediately deducible. The major axes were drawn through the ellipses and the positions of the planes of maximum tangential strain were shown by broken lines. The diagram was then reduced photographically to the same scale as the photographs from which it was derived. The result is shown in fig. 11, Pl. III.

Cleavage on the two theories.—It is now easy to draw through this quadrant of the figure representing the cross section of the cake a set of

lines which are sensibly tangent to the directions of the major axes of the ellipses, and this set of lines represents the cleavages which the cake should have according to Sharpe's theory. This result is illustrated by fig. 12, Pl. III, where, for the sake of completeness, all four quadrants are filled out. Similarly a diagram can be prepared illustrating my theory, and this is given in fig. 13, Pl. III. It will be observed that the two diagrams differ very radically and that the choice between them must be an easy one.

The system of cracks likely to occur on Sharpe's theory is made sufficiently evident by the figure just mentioned. The other theory shows an interlacing of possible fractures near the center, which is more complex and could not be entirely developed in a single cake without comminution. For this reason the most probable and important fractures are given separately in fig. 14, Pl. IV.

The surfaces which at the end of the straining process are surfaces of maximum tangential strain were never at any step of the process perpendicular to the direction of greatest linear contraction. To make sure of this, I have constructed the strain ellipses for a case in which a cylinder of ceresin pierced with a network of holes was compressed to two-thirds its original length. Fig. 6, Pl. II, shows the cross section of the strained mass, and fig. 23, Pl. V, shows lines coinciding with the direction of the major axes of the strain ellipsoids for this case. Comparison of the last diagram with that previously discussed for a strain twice as great (fig. 12, Pl. III) shows their analogy, and proves the statement made in the first sentence of this paragraph.

Cleavage actually found.—It is exceedingly difficult to give satisfactory illustrations of the cleavages actually obtained by Tyndall's experiment. In the first place, the most instructive cakes are those which go to pieces under the hammer; but then the residual flakes are too delicate to be cut across radially for photographing, and were this accomplished only a single section could be figured, whereas the observer may examine them in three dimensions. Again, when the cakes are so gently hammered as merely to crack from the edges and are subsequently separated into two or more pieces, tension ruptures are produced as well as true cleavage; but these are not distinguishable in a photograph. Especially characteristic in radial section is the way in which the cleavage meets the outer edge of the cake. Seen in cross section the thinner part of the split cake at the outer edge is shaped like one horn of a crescent moon. This characteristic is shown by every cake, and yet it is not strikingly apparent in every section illustrated in figs. 15 to 22, Pl. IV, although it is well shown in several of them. The cakes break last at the axis, and here there is most danger of tension rupture in forcing the opposite portions asunder. In two or three of the specimens illustrated, however, there is evidence

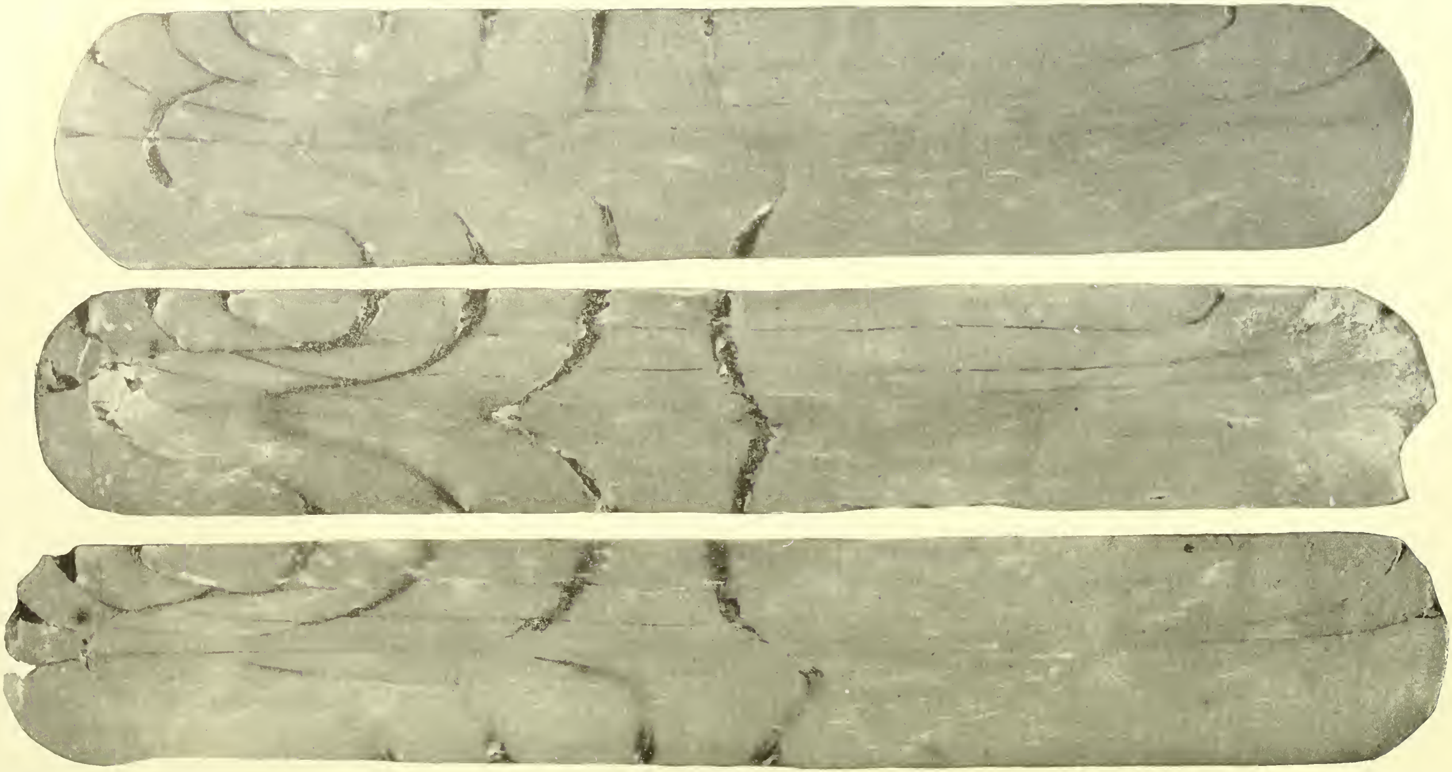

Fig. 10.

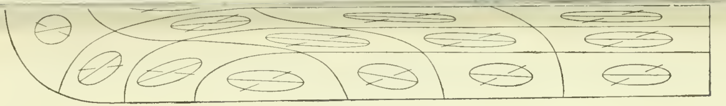

Fig. 11.

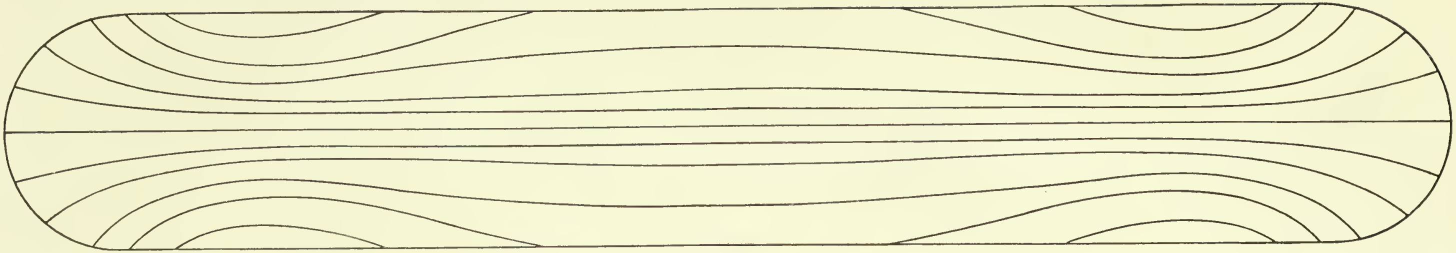

Fig. 12.

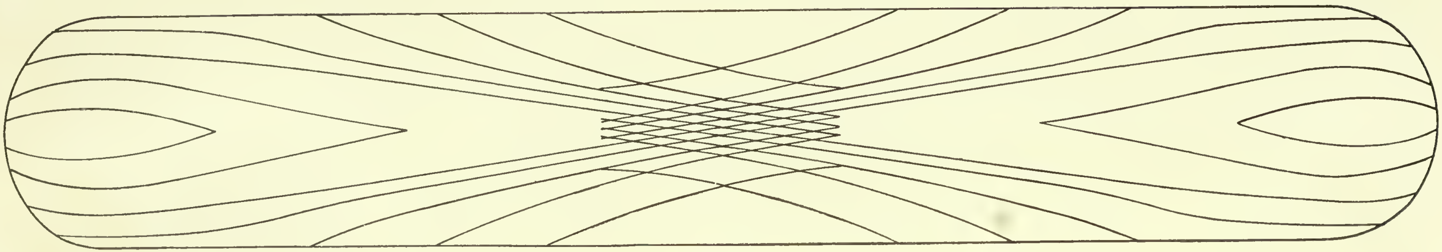

Fig. 13.

FIG. 10. THREE CAKES OF CERESIN COMPRESSED TO ONE-THIRD ORIGINAL HEIGHT, ENLARGED.
FIG. 11. DIAGRAM TO SHOW STRAIN ELLIPSOIDS IN ONE QUADRANT OF COMPRESSED CERESIN CAKE.
FIG. 12. DIAGRAM TO SHOW CLEAVAGE ON SHARPE'S THEORY.
FIG. 13. DIAGRAM TO SHOW CLEAVAGE ON BECKER'S THEORY.

of double fracture of opposite inclination at the axis (figs. 19 and 20). In figs. 18 and 21, Pl. IV, the cleavage is seen passing through the axis at an angle to the median line.

On the whole, even the photographs show a very reasonable agreement with the inference from analysis indicated as probable cleavages in fig. 14, Pl. IV. I may add that the cleavability about midway between the edge and the axis is often so perfect as to defy illustration; the flakes are frequently so thin that print can be read through them.

I must have made Tyndall's experiment a hundred times, using wax on some occasions and ceresin on others. In no case have I broken a cake which behaved as it should on Sharpe's theory, illustrated in fig. 12, Pl. III. The section of a cake most nearly in accord with that theory which I have seen is shown in fig. 22*a*, Pl. IV, and especially for that reason. Even this exceptional instance exhibits features not in agreement with that theory, while another section of the same cake (fig. 22*b*) does not at all resemble the diagram constructed for the loci of the major axes of the strain ellipsoid. The general features of the cleavage along surfaces of maximum tangential strain seem always to be recognizable when the terminations of the cylinders are plane and the compression is sufficient to produce good cleavage in the squeezed cakes, but not so excessive that the minor axes of the strain ellipsoids are reduced to almost insensible length. Evidently, in order to make any instructive comparison between the two theories, these conditions must be fulfilled.

Absence of slip cleavage.—The semitranslucency of ceresin is very advantageous for these experiments in some respects. If a cake of ceresin is examined in a strong light and at the same time partly shaded, any internal cracks can readily be detected. When cylinders are being compressed for Tyndall's experiment, between glass plates of the proper temperature, the first cracks to form seem always to be at the edge of the cake; and when the pressure is applied so gradually as to avoid this peripheral splitting I find no internal cracks unless the glass plates were too cold.

Significance of bubbles.—On the other hand, my cakes all contain numerous minute bubbles of air, carried into the mold in casting. During compression these are flattened, and are then equivalent to minute blind joints. The flattened bubbles are, of course, oriented exactly as are the strain ellipsoids. They are perfectly visible in the photographs of the dissected cakes, and comparison shows that the orientation of the bubbles is indistinguishable by the eye from that of the strain ellipsoids obtained by construction as shown in my diagram, fig. 11, Pl. III. Were the bubbles smooth internally, as they would be in a glass, it might be possible to dispense with the construction.

They are not smooth, however, and seem to be lined with minute paraffin crystals, so that their evidence, though confirmatory, is not sufficient.

I feel fully justified in asserting that there is in my experiments just described no blind jointing (Ausweichungsclivage) or slip cleavage in the directions in which cleavage actually takes place, or in directions called for by my theory of cleavage. But the bubbles tend to weaken the mass in the directions in which cleavage should occur on Sharpe's theory. Hence the weakening of cohesion, to which I attribute cleavage (along the surfaces of maximum tangential strain), must be so great as more than to counterbalance the effect of the bubbles.

The double or schistose cleavage which is called for by theory and is illustrated in fig. 13, Pl. III, is not often displayed except by the diversity in the directions of the surfaces of fracture near the axis of the cakes; but the fact that near the axis the cake may split in either of two directions shows that there are two intersecting cleavages.

Scission experiments.—The purpose of experimenting with scission was, as has been explained, to produce a simple strain which, if it could be made to lead to cleavage at all, would indicate beyond doubt whether the surfaces coincided with one of those of maximum tangential strain. In a scission one of these directions is parallel to two faces of a distorted rectangular mass, those, viz, which undergo no change of area. In this direction the same set of particles is subject to maximum tangential strain from the inception of the process to its completion, however long a time that may be. There is a second set of planes of maximum tangential strain, but as the strain increases in amplitude fresh sets of material particles continually replace one another in these latter planes, so that any one set of particles undergoes maximum tangential strain along these planes only for an infinitesimal time. Hence, either a smaller effect, or at least a different effect, will be produced on this second set of planes. If my theory is correct, cleavage is to be looked for only parallel to the planes of constant area, for reasons indicated in a note on the theory appended to this paper. With the unit shear, or when the acute angle of the distorted mass is 45°, the major axis of the strain ellipsoid makes an angle of about 32° ($\frac{1}{2}\tan^{-1}2$) with the undistorted planes. This strain is shown in fig. 31, Pl. VI.

Results for ceresin.—I never expected to get a perfectly smooth slaty cleavage by scission, for reasons stated in the appended note on my theory. Experiments on the scission of ceresin blocks are not very satisfactory. If the blocks have as high a temperature as I found best for Tyndall's experiment (where the strains for the most part are of much greater amplitude than my scission engine will produce), the blocks show no cleavage at all. At 20° C. and 0° C. cleavage sometimes results and is sometimes absent or insensible. When the cleav-

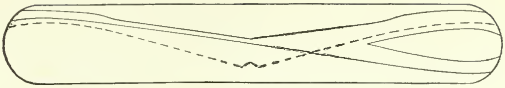

Fig. 14.

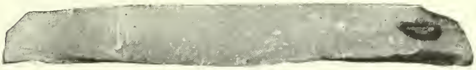

Fig. 15.

Fig. 16.

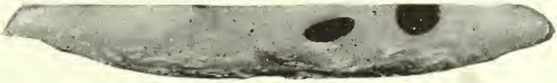

Fig. 17.

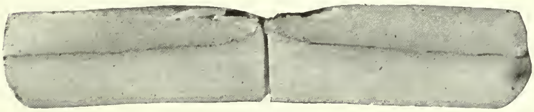

Fig. 18.

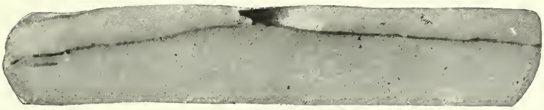

Fig. 19.

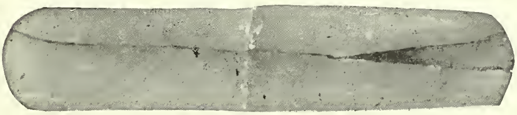

Fig. 20.

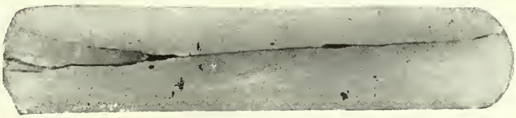

Fig. 21.

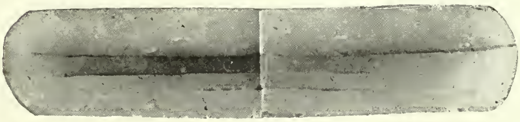

Fig. 22 *a*.

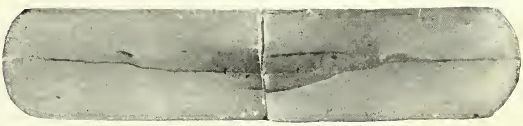

Fig. 22 *b*.

FIG. 14. PROBABLE CLEAVAGES ON BECKER'S THEORY.
FIG. 15–22. PHOTOGRAPHS OF ACTUAL CLEAVAGES.

age is a vanishing quantity or absent the mass breaks with a more or less conchoidal fracture. When there is any distinct cleavage it is parallel to the undistorted planes.

One difficulty with ceresin is that, on account of air bubbles,[a] the block does not completely and homogeneously fill the space. If it did, rupture would be impossible. In fig. 24, Pl. V, is shown an instance of rupture. The circle stamped on the block before strain is distorted to an oval which is dislocated by three tiny faults. The joints formed are exactly parallel to the undistorted plane. The truncated upper left-hand corner shows the failure to fill out the space.

In fig. 25, Pl. V, is shown a block which did not entirely fill out the space, but was not jointed like fig. 24, Pl. V. When struck on the back with a hammer it developed cracks having a tendency to parallelism with the undistorted planes. In one case, after subjecting a cake to scission, I cut off one sharp corner perpendicularly to the plane of no distortion and filled out the opposite face with a wedge, so as to reduce the block once more to a rectangle. This was again strained in the engine. After chilling it broke with some regularity. It is shown in fig. 26, Pl. V, where X is the mass which was added before the second strain.

Results for clay.—For scission clay is a far better material than ceresin, as was mentioned above, and in all the cases I have tried clay blocks subjected to unit strain and lightly burned show cleavage parallel to the planes of no distortion. A very highly instructive specimen was produced accidentally. The distorted block was dried on the water bath and then heated in an assay muffle furnace, which, however, grew hot too rapidly. The escaping water vapor burst the block into many pieces. These, fortunately, were of such sizes that it proved practicable to fit them together and restore the outlines of the block. This specimen is illustrated by a photograph (fig. 27, Pl. V). The ellipse is visible and not faulted, and the cleavage is manifestly parallel to the planes of no distortion.

Lessons drawn.—The experiments described above, which constitute a study in plasticity, appear to me to demonstrate that true cleavage (wholly free from blind joints, or Ausweichungsclivage) can be produced both in ceresin and in clay. Burning the clay does not obliterate the cleavage. The cleavage does not coincide even approximately with the direction of the major axes of the strain ellipsoids. Neither does the cleavage correspond to the position of the major axes of the strain ellipsoids at any previous stage of the strain. On the other hand, the orientation of the cleavage does correspond to the position

[a] It might be better to prepare in another way blocks of ceresin for scission. The melted mass might be very gradually cooled, with very gentle stirring, in a flat-bottomed pan, and there allowed to solidify without pouring. Then blocks could be sawn out of the mass and planed to fit the engine. Such blocks would be free from bubbles. The experiments with clay are so satisfactory that I did not try this method.

of the surfaces of maximum tangential strain. Cakes of ceresin linearly compressed exhibit and elucidate slaty cleavage near their edges. Toward their axes of symmetry they show and explain the double or multiple cleavage so characteristic of the crystalline schists. This last important phenomenon is almost unintelligible on Sharpe's theory, for if the greatest linear contraction were perpendicular to one set of cleavages in the schists, it could not also be perpendicular to the other. If it be suggested that the two (and sometimes four) cleavages were successively impressed on the schist, the answer is that observation is inconsistent with this explanation, the distribution of minerals and their mutual relations contradicting the idea. The evidence presented shows that rupture and cleavage follow the same surfaces, cleavage being due, so far as can be told, to weakened cohesion—a state of things in absolute accord with Daubrée's experiments and Heim's observation that Ausweichungsclivage and cleavage without rupture are sometimes visible in the same slide and are parallel to each other.[a] The effect of pressure in the direction of greatest linear contraction, on the other hand, is only to force molecules closer together in the line joining their centers. How this approach of molecules to one another might increase their cohesive attraction I can understand; but how it could weaken it is, to me, a mystery. So far as I know, no theory of molecular attraction has been formulated which would account for such a weakening.

The deformation of ceresin implies only a trifling expenditure of energy or a minute evolution of heat. When firm rocks are deformed, especially igneous rocks, the amount of work done or of heat evolved must be very great. That such deformation should be accompanied by the genesis of secondary minerals is in accordance with all the results of the study of metamorphism. Now, experiments of my own, not yet published, show conclusively that crystals tend to grow in the direction of least resistance, as might indeed be assumed, although they exert a linear force in any direction. Secondary minerals in a slate originating in a firm rock will thus tend to develop chiefly in the direction of cleavage. It is not improbable that the secondary development of mica on cleavage planes may further facilitate the cleavage to which it owes its existence, much as slickensides on a faulted surface facilitate further faulting. If the mica were assumed as the origin of the cleavage, it would be necessary to show how it could be generated and oriented in planes independent of the stratification without obliterating

[a] Mechanismus der Gebirgsbildung, vol. 2, 1878, pp. 56 and 59. Heim attributes cleavage to movements (Ausweichungen) of the mass perpendicularly to the direction of pressure. It is evident that he refers to *relative* movements of adjacent portions of the mass, or what I call tangential movements. These necessarily imply forces locally inclined to the direction of the relative motion, for, in general, in any solid or in any hyperviscous liquid strained at a finite rate, a tangential displacement implies a force containing a tangential component. Heim insists that rupture and cleavage, when due to one force, are parallel, a point in which I agree with him. On the other hand, master joints and false cleavage occur characteristically at angles of more than 45° to the slaty cleavage.

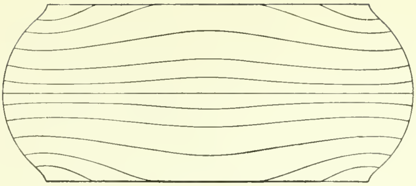

Fig. 23.

Fig. 24.

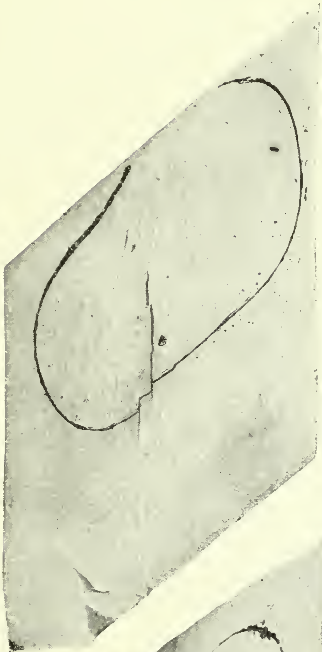

Fig. 25.

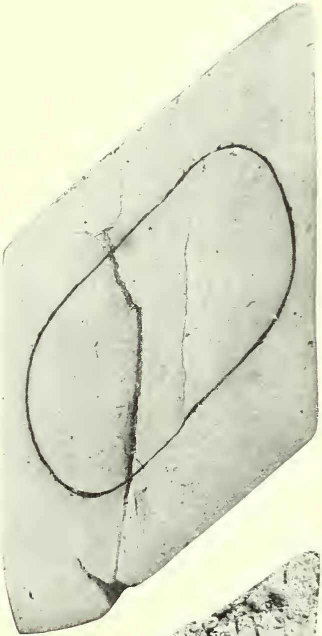

Fig. 26.

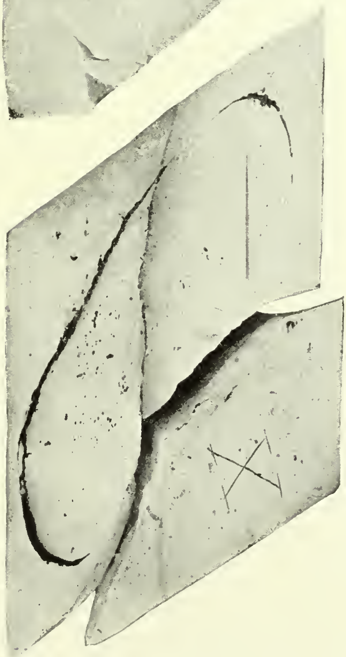

Fig. 27.

FIG. 23. LOCI OF MAJOR AXES OF ELLIPSOIDS CORRESPONDING TO FIG. 6.
FIG. 24. BLOCK OF CERESIN FAULTED BY SCISSION.
FIG. 25. BLOCK OF CERESIN SHOWING CLEAVAGE DUE TO SCISSION.
FIG. 26. BLOCK OF CERESIN SHOWING CLEAVAGE DUE TO DOUBLE SCISSION.
FIG. 27. BLOCK OF CLAY SHOWING CLEAVAGE DUE TO SCISSION.

the stratification, as well as why grit beds in slate show cleavage. This, it seems to me, has never been successfully done.

The experiments made would indicate that the order of linear compression in slates, supposing their volume inalterable, is about in the ratio of 1 to 3 or 4, and that it is accompanied by large slides or tangential strains; but the degree of strain needful to produce cleavage probably depends both on the nature of the material and on the rate of straining. From field observations I suspect that a compression of one-half sometimes suffices.

I infer that a slate belt is equivalent to a distributed fault or a step fault with infinitesimal steps, whose total displacement is of the same order as the thickness of the slate. The direction of this faulting (according to the results reached in my former discussion) is given by the intersection of the cleavage plane with a plane perpendicular to the grain of the slate, and is therefore ordinarily not greatly inclined to the horizon. Were the grain vertical it would indicate horizontal faulting. The major axis of the strain ellipsoid lies in the plane which is perpendicular to the grain of the slate but at a considerable angle to the cleavage.

The force to which cleavage is due lay in this same plane at an angle to the cleavage which would be zero if the strain were unalloyed scission, barely conceivable in nature, and finite for all other strains. There is no simple relation between the direction of the force producing strain and the directions of the axes of the strain ellipsoid for any case of rotational strain. In the case of slate the direction of the force lay within the acute angle between the direction of greatest linear compression (or the smallest axis of the strain ellipsoid) and the cleavage. In the case of symmetrically developed double or multiple schistosity, not infrequent in the crystalline schists, rotation was absent and the direction of force coincided with that of the smallest axis of the strain ellipsoid bisecting the obtuse angle between the cleavages. It appears probable, from the experiments, that the angle between the slaty cleavage and the local direction of the force to which it is due may vary within wide limits.

For other consequences of the theory confirmed by the experiments here described I must refer to my former memoir.[a]

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[a] It is easier to test the experimental results reached in this paper, now that it is written, than to examine microscopically even a very small suite of rocks. The following method of verification is suggested: Any young student or handy janitor can prepare a set of cylinders of ceresin cast at the lowest practicable temperature, with flat ends, and of a diameter equal to the length. Keep ten or twelve such cylinders in a thermostat over night at 35° C. Compress them, three at a time, between heavy, somewhat warm glass plates in a copying press to one-third of their original length, so slowly as not to burst the edges. Put the cakes in ice and salt for an hour or more. Then cut one or more of the cakes in two on a plane which includes the axis. Examination of the minute bubbles should show whether my diagram of the distribution of strain ellipsoids adequately expresses the facts. If it does so, the figures exhibiting the alternative cleavage, according to Sharpe's theory or mine, must also be correct. Split the rest of the cakes by striking them edgewise with a hammer, and compare the cleavage with the diagrams.

MATHEMATICAL NOTES.

NOTE ON THE THEORY OF SLATY CLEAVAGE.

The theory of rupture of rock masses under pressure which I have propounded[a] is that fracture occurs along planes of maximum tangential strain, or, as it is also called, of maximum slide. Cleavage I regard as due to a weakening of cohesion, antecedent to rupture, on these same planes of maximum slide, the effects being influenced by viscosity, although the direction is independent of viscosity. For the full development of this theory the reader must be referred to the former memoir, just cited, but some essential features should be included here.

The planes of maximum tangential strain in a homogeneously strained mass are readily found. Their position relative to the major axis of the strain ellipsoid is independent of the cubical compression to which the mass may have been subjected, and of any rotation which the axes may have undergone relatively to the elements of mass. These positions are therefore dependent only on the two pure undilational shears, at right angles to each other, which determine the relative magnitude of the axes of the strain ellipsoid. These two shears may be separately considered.

The first problem is, then, to find the planes of maximum tangential strain in an irrotational shear ellipsoid. In this ellipsoid the section containing the greatest and least axes is an ellipse of the same area as the corresponding great circle of the original sphere. If the radius of the circle is taken as unity, the major axis of the ellipse may be called α (the "ratio of shear") and the minor axis will be $1/\alpha$, so that all lines in the ellipse parallel to the major axis of the ellipse exceed their original length in the ratio α, and all lines parallel to the minor axis have been reduced in length in the ratio $1/\alpha$.

In the circle draw any parallelogram, for instance one with its center at the center of figure, and from the center draw two radii parallel to the sides of the parallelogram, making angles ϑ and ϑ_1 with the major axis, and meeting the circle at points $x\ y$ and $x_1\ y_1$, as shown in fig. 28, Pl. VI. Then in the ellipse there will be a corresponding parallelogram and set of points which may be indicated by the same letters primed.

[a] Finite homogeneous strain, flow, and rupture of rocks: Bull. Geol. Soc. America, vol. 4, 1893, pp. 13–90.

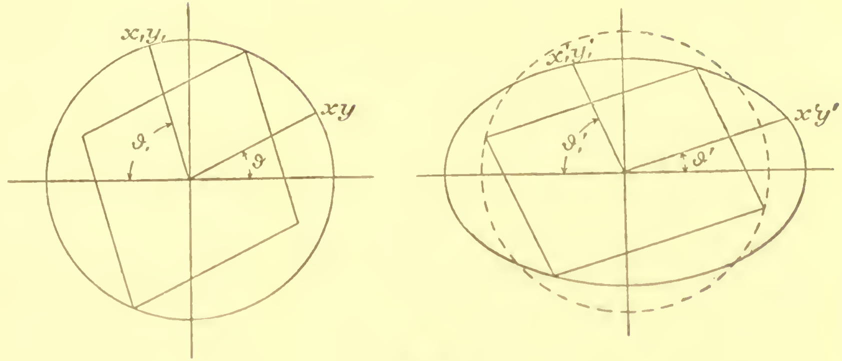

Fig. 28.

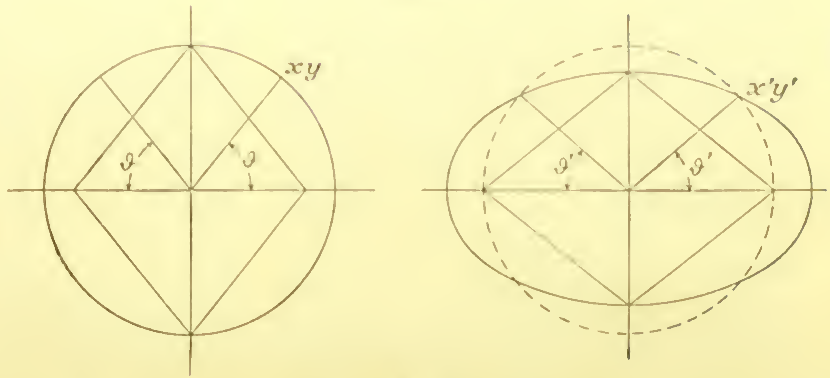

Fig. 29.

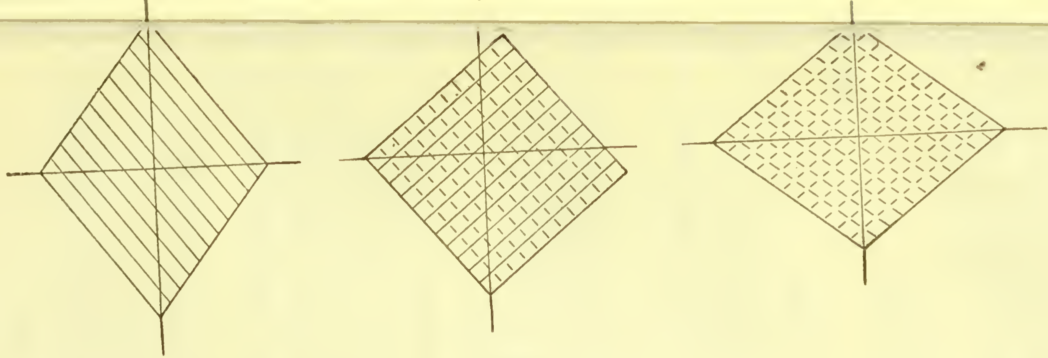

Fig. 30.

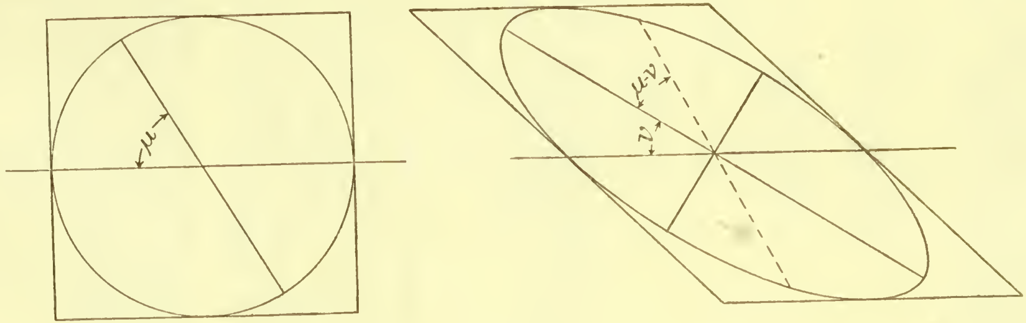

Fig. 31.

FIG. 28. DIAGRAM OF SLIDE, BUT NOT MAXIMUM SLIDE.
FIG. 29. DIAGRAM OF MAXIMUM SLIDE.
FIG. 30. PURE SHEAR COMPOUNDED OF TWO SCISSIONS.
FIG. 31. SCISSION AND ANGLE OF ROTATION $\mu-\nu$.

Now slide will change the angles of the original parallelogram, or the inclination of the radii in the circle, and the greater this change the greater the slide. If, therefore, the maximum slide is sought, the deflection of each of the two radii must be a maximum, and symmetry shows that ϑ and ϑ_1 must be equal. Hence the problem reduces to finding the radius in the circle which experiences the greatest change of direction through the strain. This is very easy, for by definition (see fig. 29, Pl. VI),

$$x'=\alpha x;\ y'=y/\alpha;$$

$$\tan\vartheta=y/x;\ \tan\vartheta'=\frac{y'}{x'}=\frac{\tan\vartheta}{\alpha^2};$$

so that

$$\tan(\vartheta-\vartheta')=\frac{\tan\vartheta\,(\alpha^2-1)}{\alpha^2+\tan^2\vartheta},$$

which has its maximum value when

$$\tan\vartheta=\alpha;\ \tan\vartheta'=1/\alpha.$$

Thus the maximum value of

$$\tan(\vartheta-\vartheta')=\frac{\alpha-\alpha^{-1}}{2}.$$

The total change of the angle of the original rhomb is measured by twice this quantity, and $\alpha-\alpha^{-1}$ is known as the "amount of shear."[a]

In polar coordinates the equation of the ellipse is

$$\frac{\cos^2\vartheta'}{\alpha^2}+\alpha^2\sin^2\vartheta'=\frac{1}{r^2},$$

and when $\alpha=\cot\vartheta'$, evidently $r=1$, so that the directions in which slide is a maximum for a single shear are those in which the radii have preserved their original length. In other words, if the circle is superposed upon the ellipse, the intersections of the two curves are the extremities of the radii in question. Hence, also, the planes of maximum tangential strain in the shear ellipsoid are the circular sections.

This subject can also be profitably considered from another point of view, that of the stresses involved. If a rod is subjected to a finite tensile load Q, I have shown[b] that the resultant stress (force per unit area), R, the normal stress, N, and the tangential stress, T, in one component shear, may be written as follows:

$$R^2=\frac{Q^2}{9}\left(\alpha^2\sin^2\vartheta'+\frac{\cos^2\vartheta'}{\alpha^2}\right).$$

$$N=\frac{Q}{3}\left(\alpha\sin^2\vartheta'-\frac{\cos^2\vartheta'}{\alpha}\right).$$

$$T^2=\frac{Q^2}{9}\left(\alpha+\frac{1}{\alpha}\right)^2\sin^2\vartheta'\cos^2\vartheta'.$$

---

[a] It would be much better to measure shear by the quantity $\frac{\alpha-\alpha^{-1}}{2}$, and to call this the *amplitude* of shear.

[b] The finite elastic stress-strain function: Am. Jour. Sci., 3d ser., vol. 44, 1893, pp. 337-356.

Comparing this value of R^2 with the polar equation of the ellipse, it appears that for any value of ϑ'

$$rR = \pm Q/3,$$

so that the resultant load or initial stress, or stress into final area, is the same on any section. Now, for the circular section, or $\tan\vartheta' = 1/\alpha$, $N=0$, so that the entire load is tangential, and although the tangential stress (as is well known) is greatest for $\vartheta' = 45°$, the tangential load, $\pi r\ T$, is greatest for $\tan\ \vartheta' = 1/\alpha$, and then becomes $\pi T = \pi Q/3 = \pi R$. Thus, according to the theory here set forth, rupture and cleavage are determined by maximum tangential load, not by maximum tangential stress.

If a second shear is applied to the mass in a plane at right angles to the first, the effect in the plane of the first shear is only to reduce the height without altering the breadth. Consequently no further slide is produced in the plane of the first shear and no further tendency to the impairment of cohesion or to its dissolution exists. On the other hand, the angle of the planes of maximum tangential strain to the major axis of the ellipse is modified. If the final angle made by these planes with the major axis A, is ω, and if B and C are the other axes of the ellipsoid (A> C> B) it is easy to see[a] that in the plane AB

$$\tan^3\omega = B^2/AC;$$

or if the mass is incompressible, so that $ABC=1$,

$$\tan\ \omega = B.$$

From the point of view of cleavage and rupture, the inner mechanism of a pure shear is important. Suppose the rhomb shown in fig. 30a, Pl. VI, to be divided into an infinite number of equal strips, each of length equal to a side, and that these be slid over one another as the cards of a pack can be slipped. Then the resulting figure may be a square, shown at b. Divide this square anew into strips at right angles to the former divisions and shift these new strips as shown in fig. 30c, Pl. VI. The result of the double process will be a rhomb identical with that due to pure shear, shown in the preceding diagram, fig. 29, Pl. VI.[b]

Now, not only is shear produced by this mechanism, but it seems impossible to devise any other mechanism by means of which it can be produced. The significant point is that action is almost confined to planes parallel to the sides of the rhomb; elsewhere the only relative movement which occurs is mere approach or separation of molecules on lines joining their centers. It is conceivable that the lengthen-

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[a] Bull. Geol. Soc. America, vol. 4, 1893, pp. 34 and 22.

[b] The two component strains are scissions, and it is susceptible of easy algebraic proof that if two scissions of equal amplitude are superposed in such a manner as to produce pure shear the two scission planes must be at right angles to each other. If the ratio of the resultant shear is α, the ratio of shear in each of the scissions must be $\sqrt{\alpha}$.

ing of elements parallel to the major axis should cause rupture perpendicular to this direction, or in planes parallel to the plane of the mean and minor axis. It is also true that when short cylinders are subjected to linear compression, their edges sometimes split by tension meridionally or in planes perpendicular to the plane of the two greater axes of the strain ellipsoid, and that the blocks sometimes yield along planes of maximum tangential strain. But that the approach of molecules along the smallest axis, on lines joining their centers should tend to weaken their cohesion and produce cleavage perpendicular to the smallest axis seems to me most improbable from a mechanical point of view, and I have found no experimental evidence of such an effect. Such an effect, however, is implied in Sharpe's theory of cleavage.

In a pure shear the lines of maximum tangential strain do not coincide throughout the strain with the same material particles. The first particles to undergo this strain stood originally at 45° to the line of pressure, while those which ultimately underwent maximum tangential strain originally lay at angles of less than 45° to the line of pressure—i. e., $\tan^{-1} 1/\alpha$. These lines of strain thus rotate through wedges of the strained mass. The axes of the ellipsoid, however, in any so-called pure strain coincide with the same sets of particles throughout, and maintain an invariable direction.

In rotational strains, on the other hand, the axes of the ellipsoid wander, so that successive sets of particles become axial. This rotation of the axes affects also the rotation of the lines of maximum slide. The axial rotation adds to the rotation of one set of lines of maximum slide and diminishes the rotation of the other set. Hence, in rotational strain one set of sliding planes wanders through the mass faster than the other. Consequently, also, on one side of the minor axis a given radial layer of particles is subjected to maximum tangential strain for a longer time than the corresponding layer on the other side. The angle of rotation of a strain is the angle between either axis of the strain ellipsoid and the line which passed through the same set of particles before strain began.

The extreme case of rotational strain, and the most important one, is scission (fig. 31, Pl. VI). In scission the rotation of the axes of the strain ellipsoid exactly compensates for the rotation which one set of planes of maximum strain would have in an irrotational strain of equal amount. Hence, in scission this latter set of planes does not rotate at all, or, in other words, the same set of material particles is subject to maximum tangential strain from the inception of strain to its conclusion. The other set of planes rotates through the mass just twice as quickly as it would if the strain were "pure."

Now, all real matter is viscous, and a solid displays its viscosity by yielding gradually to force up to a certain limit. A mass to which force is applied for a brief time interval resists deformation not only in virtue

of its "rigidity,"[a] but in virtue of its viscosity also. Hence, in the case of rotational strains in viscous solids, those layers of particles through which the planes of maximum tangential strain move more slowly will experience greater permanent deformation than the layers on the other side of the minor axis. These last, when the difference of rotation is considerable, will either recover elastically or rupture like a brittle body, thus giving rise to master joints and false cleavage.

Hence, according to the theory here propounded, slaty cleavage will result, in suitable material, from rotational strains, and will be found on the side of the least axis of the strain ellipsoid from which rotation takes place. In other words, it will occur at an angle to the major axis, the tangent of which is the third root of B^2/AC, but at only one of the angles so defined. It does not seem probable, from a theoretical point of view, that scission by itself will produce relatively perfect or smooth cleavage. If the cleavage due to scission alone had a certain roughness, and if the mass were further subjected to a linear compression that would reduce it to a fourth of its original thickness, this roughness would also be reduced to a fourth, or the cleavage would be four times as smooth as in simple scission. Furthermore, the conditions under which scission alone is produced must be extremely exceptional among rocks. All deformations, however, can be resolved into scissions; so that, if it were to be said that cleavage is due to strains compounded of scissions, this would merely be equivalent to asserting that cleavage is due to deformation.

NOTE ON INSCRIPTION OF AN ELLIPSE IN A PARALLELOGRAM.

This problem is easily solved algebraically if, for example, the parallelogram is derived from a square by displacements. It is thus treated in my former memoir on homogeneous strain. For such discussions as have been offered in this paper it is convenient to use a graphical method.

Let the sides of the parallelogram in fig. 32, Pl. VII, be $2a'$ and $2b'$, the value of a' being greater than that of b', and let the acute angle between the sides be ψ. Draw through the center of the figure lines parallel to the sides, then these lines will be conjugate diameters of the inscribed ellipse. By the properties of conjugate diameters these are connected with the axes a and b by the two relations,

$$(a \pm b)^2 = a'^2 + b'^2 \pm 2a'\,b' \sin \psi.$$

[a] Rigidity is resistance to change of shape when the force is so gradually applied that viscosity does not come into play. Rubber is a rigid body with a low modulus of rigidity. Rigidity in this technical sense is a property common to all solid bodies.

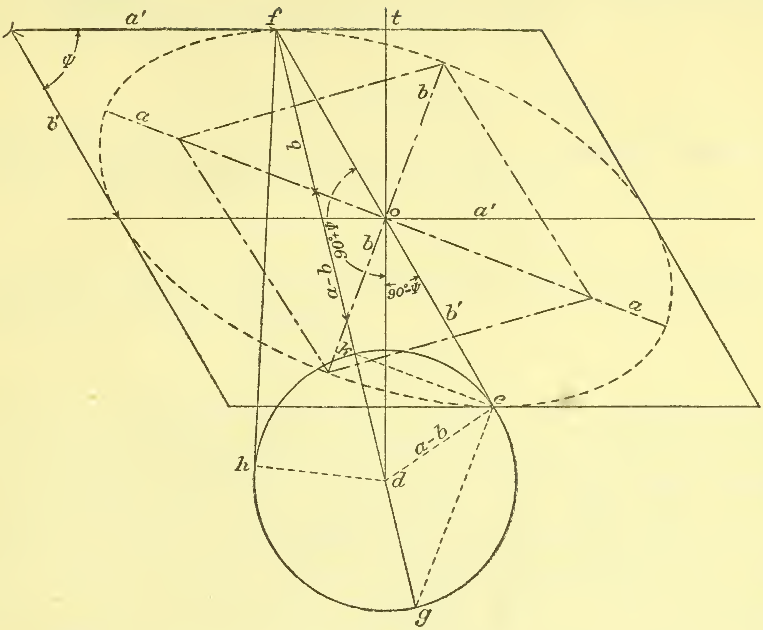

Fig. 32.

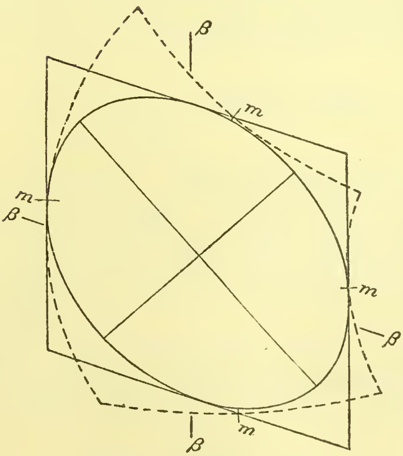

Fig. 33.

FIG. 32. INSCRIPTION OF ELLIPSE IN PARALLELOGRAM.
FIG. 33. APPROXIMATE INSCRIPTION OF ELLIPSE IN CURVILINEAR QUADRILATERAL.

From the center, o, draw a line od perpendicular to a' and of length a'. Connect the foot of b' (or the point e) with d. Then in the triangle ode the angle doe is $90°-\psi$, and therefore

$$\overline{de}^2=a'^2+b'^2-2a'\,b'\sin\psi=(a-b)^2.$$

Again draw from d to the top of the shorter conjugate diameter the line df. Here the angle fod is $90°+\psi$, and hence

$$\overline{df}^2=a'^2+b'^2+2a'\,b'\sin\psi=(a+b)^2.$$

With de as radius, describe a circle about d cutting fd at k and prolong fd to g. Then, evidently,

$$fg=2a;\ fk=2b;$$

so that the magnitude of the axes is known. To find their position, draw through o lines parallel to the chords ek and eg. The line fd will then be divided as in the ordinary construction of the ellipse, founded on the theorem that if two fixed points, on a right line, are constrained to move on rectangular axes, the curve generated by any other point on the line will be an ellipse whose greatest and least diameters lie in the given rectangular axes. The construction shows that f is a point on the ellipse. There are two positions of the axes found which are compatible with this condition, but only one which is compatible with the further condition that ft shall be tangent to the curve. Hence a is parallel to ek and b is parallel to eg.

I find that a closely analogous construction was given by Mannheim, in 1857.[a] That here presented has certain advantages.

To find the mean radius of the ellipse, draw the line fh tangent to the circle. Then

$$\overline{fh}^2=(a+b)^2-(a-b)^2=4ab;$$

$$fh=2\sqrt{ab},$$

or twice the radius required. Also the angle fdh is the acute angle between mean radii of the ellipse. If the length $\sqrt{ab}$ is set off on a, and lines are drawn from this point to the extremities of the minor axis, they will be parallel to the mean radii and correspond to the sides of the rhomb in fig. 29.

If only the position of the axes in the rhomb is wanted, it can conveniently be found as follows without determining the magnitude of the axes. Let ϑ be the angle between a and a'; then

$$\tan 2\vartheta=\frac{\sin 2\psi}{\cos 2\psi+a'^2/b'^2}.$$

To prove this equation, write the equation of the ellipse referred to its conjugates as axes in the well-known form $x'^2/a'^2+y'^2/b'^2=1$. If x and

[a] See Williamson's Diff. Calc., 9th ed., p. 374.

y are abscissa and ordinate of a point on the ellipse referred to rectangular axes, one of them containing a',

$$y'=y/\sin\psi;\ x'=x-y\tan\psi.$$

Substitution gives an equation of the ellipse in rectangular coordinates equivalent to

$$a_{11}x^2+2a_{12}xy+a_{22}y^2=1,$$

and it is well known that

$$\tan 2\vartheta=\frac{2a_{12}}{a_{11}-a_{22}},$$

which gives the value stated above. This value may be old, but I do not recall having seen it.

When it is desirable to draw the strain ellipse characteristic of the central point of a curvilinear quadrangle representing a small portion of a strained mass, a rational method which can not be very erroneous is as follows: In fig. 33, Pl. VII, at the central points, m, of two opposite curved sides draw tangents and from their intersection a line, β, bisecting the angle between the tangents. Proceeding in the same way with the other two curvilinear sides, find a second bisectrix. Next draw through the middle points, m, of the curvilinear sides lines parallel to the bisectrices. These parallels form a parallelogram in which, as shown above, it is easy to inscribe an ellipse, which, however, may not make an exact contact with the curved sides, though it usually approaches closely to contact.

To examine the legitimacy of this construction, suppose that not merely the sides were given but a large number of intermediate curves. Then at the center there would be a small parallelogram whose sides would have directions intermediate between the tangents at the points m. Evidently the simplest hypothesis is that the directions of these sides would each be the mean of those of the two opposite tangents, and this assumption is made in the construction. Though the spacing of the intermediate curves would in general vary, it is perfectly legitimate to assume that for small distances the spacing would vary linearly, so that in the central parallelogram points corresponding to m would lie at distances apart which are simply proportional to those of these points on the curvilinear quadrangle. No further assumption is made in the construction, and I can see no doubt that it is sufficiently accurate for any such purpose as that to which it has been applied in the foregoing paper.

NOTE ON COMPUTATION OF TAN ω.

Let a, b, c be the axes of the strain ellipsoid, c being vertical to the plane of the diagram, and let $r^3=abc$. If x is the original distance of

a point from the axis of the cylinder and x' the final distance of the same point from the axis of the compressed cake,

$$\frac{c}{r}=\frac{2\,\pi\,x'}{2\,\pi\,x}=\frac{x'}{x},$$

$$\frac{rx'}{x}=\frac{r^3}{ab}=c,$$

$$\pi\,ab=\pi\,r^2\,\frac{x}{x'},$$

the area of the ellipse. In preparing fig. 11, Pl. III, I found that the ellipses obtained by construction agreed very fairly with the last formula. The differences seemed due to slight inaccuracy in reproducing the network of curved lines in the dissected cakes, fig. 10, Pl. III.

The angle ω is given by

$$\tan^2\omega=\frac{b^2}{r^2}=\frac{x}{x'}\,\frac{b}{a},$$

or if A and B are the axes found by construction, and if it is assumed that $B/A=b/a$,

$$\tan\,\omega=\sqrt{\frac{x}{x'}\,\frac{B}{A}}=\frac{b}{r},$$

and

$$\sqrt{\frac{x}{x'}\,\frac{A}{B}}=\frac{a}{r}.$$

In fig. 11, Pl. III, the ellipses are plotted from the computed axes, the ellipticity and orientation being derived from the construction. Only a very minute examination would distinguish this from a diagram depending solely on construction.

NOTE ON VOLUME CHANGES IN THE FORMATION OF SLATE.

In some theories of slaty cleavage, cubical compressibility of the mass is invoked to explain certain phenomena; but it seems very doubtful to me whether this can play any notable part in the process of slate making. In plastic deformation a mass must first be strained to the elastic limit, at which there is a cubical compression corresponding to the intensity of the force (or to the stress) needed to produce this strain. When the stress is increased above this limit plastic deformation sets in and is attended by no further change of volume. Thus

the change of volume is a function of the stress at the elastic limit, and the cubical compression is one-third of this stress divided by the modulus of compressibility.

Now, slates are in part derived from structureless shales which, at any considerable distance beneath the surface, are moist. Their compressibility must be intermediate between that of water and that of their mineral components. Other slates, again, are produced from firm rocks like granite. The moduluses of compression of shale and granite are not known; that of water is 21×10^6 grams per square centimeter; that of quartz, 387×10^6 (Voigt), and that of glass from 347×10^6 to 437×10^6 (Everett). Again, the breaking strain of concrete under compressive loads is from 80,000 grams per square centimeter upward, and that of granite is 1,006,000 (v. Bach). Of course the elastic limit is lower than the breaking strain. Now, if shale is as strong as an inferior concrete and as compressible as water, the cubical compression at the elastic limit will be

$$\frac{.08\times10^6}{3\times21\times10^6}=.0013;$$

and if granite is as compressible as quartz, the cubical compression when it begins to flow will be

$$\frac{1.006\times10^6}{3\times387\times10^6}=.00087.$$

In each case the cubical compression (which is three times the linear compression) turns out nearly one-tenth of 1 per cent, a quantity the evidences of which it would be very difficult to trace in the field. It seems to me that the change in volume of shales and granites must be of this order, and that, for most purposes, they may be regarded as incompressible.

Doubtless the metamorphism and hydration of slate is attended by changes in density; but densities of 2.60 to 2.80 appear to include some clays, all the clay slates and phyllites of which I have a record, and the more typical granites. These densities thus afford no evidence that considerable increase in density attends the development of cleavage.

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